271. Proposed by E. B. ESCOTT, Ann Arbor, Mich.

In the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2\frac{d^5y}{dx^5}-5\left(\frac{d^2y}{dx^2}\right)\frac{d^3y}{dx^3}\cdot\frac{d^4y}{dx^4}+\tfrac{40}{9}\left(\frac{d^3y}{dx^3}\right)^3=0,$$

show that there is an integrating factor of the form $\left(\dfrac{d^2y}{dx^2}\right)^n$, and integrate the equation.

Solution by the PROPOSER, and LEVI S. SHIVELY, Mt. Morris College, Mt. Morris, Ill.

When the equation is multiplied by $(d^2y/dx^2)^n$, if it is an exact equation, its first integral is

$$\left(\frac{d^2y}{dx^2}\right)^{n+2}\frac{d^4y}{dx^4}+\frac{40}{9(n+1)}\left(\frac{d^2y}{dx^2}\right)^{n+1}\left(\frac{d^3y}{dx^3}\right)^2=c_1\ldots(2).$$

Differentiating this equation and comparing with original equation, we find that $n=-\frac{11}{3}$ and $-\frac{13}{3}$.

In (2), put $n=-\frac{11}{3}$. The equation is exact and its integral is

$$\left(\frac{d^2y}{dx^2}\right)^{-\frac{5}{3}}\frac{d^3y}{dx^3}=c_1x+c_2.$$

Integrating again,

$$-\tfrac{3}{2}\left(\frac{d^2y}{dx^2}\right)^{-\frac{2}{3}}=\frac{c_1}{2}x^2+c_2x+c_3.$$

Solving for d^2y/dx^2, and changing the constants,

$$\frac{d^2y}{dx^2}=\frac{1}{(c_1x^2+2c_2x+c_3)^{\frac{3}{2}}}.$$

Integrating twice, we have

$$y=\frac{1}{c_1c_3-c_2^2}\sqrt{(c_1x^2+2c_2x+c_3)}+c_4x+c_5.$$

Also solved by G. B. M. Zerr, and V. M. Spunar.

272. Proposed by CLARENCE OHLENDORF, Chicago, Ill.

Find $\int\log_e\tan^{-1}xdx$.

Solution by J. SCHEFFER, A. M., Hagerstown, Md., and C. N. SCHMALL, New York City.

Putting $x=\tan y$, we have

$$\int\log\tan^{-1}xdx=\int\log y\frac{dy}{\cos^2y}=\int\log y\sec^2ydy=\log y-\tan y-\int\frac{\tan ydy}{y}.$$